lulumath

Transformations of Functions IV - Horizontal Stretches, Compressions, and Reflections

Video Notes

Video Link

Transformations of Functions IV - Horizontal Stretches, Compressions, and Reflections

Background Knowledge:
- Parent Functions, Parts I and II

Consider some parent functions (including quadratic, square root, absolute value, reciprocal, and exponential) and determine what effect k has on $y = f(kx)$, for k = 2, ⅓, -4, -⅕.

$$y = f(kx)$$

output ↑   ↑ affects input

output stays same

Square Root Functions:

Parent Function: $f(x) = \sqrt{x}$

$y = f(2x)$: $k = 2$ $y = \sqrt{2x}$ → horizontal compression (s.f. = ½)

$y = f(\frac{1}{3}x)$: $k = \frac{1}{3}$ $y = \sqrt{\frac{1}{3}x}$ → horizontal stretch (s.f. = 2)

$y = f(-4x)$: $k = -4$ $y = \sqrt{-4x}$ → horizontal compression (s.f. = ¼) AND reflection in y-axis

$y = f(-\frac{1}{5}x)$: $k = -\frac{1}{5}$ $y = \sqrt{-\frac{1}{5}x}$ → horizontal stretch (s.f. = 5) AND reflection in y-axis.

$-5x$	$-\frac{1}{4}x$	$3x$	$\frac{1}{2}x$	x	$f(x)$
0	0	0	0	0	0
-5	-¼	3	½	1	1
-20	-1	12	2	4	2
-45	-2.25	27	4.5	9	3
-80	-4	48	8	16	4

y-values stay

* We are multiplying the inputs (x) by the reciprocal of k ($\frac{1}{k}$).

Quadratic Functions:

Parent Function: $f(x) = x^2$

$y = f(2x)$: $K = 2$ $\quad y = (2x)^2 \quad$ s.f. $= \frac{1}{2} \rightarrow$ horizontal compression

$y = f(\frac{1}{3}x)$: $K = \frac{1}{3}$ $\quad y = (\frac{1}{3}x)^2 \quad$ s.f. $= 3 \rightarrow$ horizontal stretch

$y = f(-4x)$: $K = -4$ $\quad y = (-4x)^2 \quad$ s.f. $= -\frac{1}{4}$ ⟶ horizontal compression
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ⟶ reflection in y-axis

$y = f(-\frac{1}{5}x)$ $K = -\frac{1}{5}$ $\quad y = (-\frac{1}{5}x)^2 \quad$ s.f. $= -5$ ⟶ horizontal stretch
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ⟶ reflection in y-axis

Summary:

$$y = f(Kx)$$

K determines:
- horizontal stretches and compressions
  scale factor $= \frac{1}{K}$

not affected by K's sign

- horizontal reflections (reflections in the y-axis)

★ $\left|\frac{1}{K}\right| > 1 \rightarrow$ horizontal stretch

★ $0 < \left|\frac{1}{K}\right| < 1 \rightarrow$ horizontal compression

★ $K < 0$ (negative) $\rightarrow$ horizontal reflection (reflection in y-axis)