lulumath

Transformations of Functions III - Vertical Stretches, Compressions, and Reflections

Video Notes

Video Link

Transformations of Functions III - Vertical Stretches, Compressions, and Reflections

Background Knowledge:

- Parent Functions, Parts I and II

Consider some parent functions (including quadratic, square root, absolute value, reciprocal, and exponential) and determine what effect a has on $y = af(x)$, for $a = 2, \frac{1}{3}, -4, -\frac{1}{5}$.

$$y = a \cdot f(x)$$

y: new output

$f(x)$: old output

x: input doesn't change

Absolute Value Functions:

Parent Function: $f(x) = |x|$

$y = 2f(x)$: $y = 2|x|$

$y = \frac{1}{3}f(x)$: $y = \frac{1}{3}|x|$

$y = -4f(x)$: $y = -4|x|$

$y = -\frac{1}{5}f(x)$: $y = -\frac{1}{5}|x|$

x	$f(x) = \|x\|$	$2f(x)$	$\frac{1}{3}f(x)$	$-4f(x)$	$-\frac{1}{5}f(x)$
-3	3	6	1	-12	-3/5
-2	2	4	2/3	-8	-2/5
-1	1	2	1/3	-4	-1/5
0	0	0	0	0	0
1	1	2	1/3	-4	-1/5
2	2	4	2/3	-8	-2/5
3	3	6	1	-12	-3/5

input is the same

REFLECTION comes from negative values

Quadratic Functions:

Parent Function: $f(x) = x^2$

$y = 2f(x)$: $y = 2x^2$ ↗ all outputs are doubled, ∴ the graph is vertically stretched (scale factor = 2)

$y = \frac{1}{3}f(x)$: $y = \frac{1}{3}x^2$ → vertically compressed (scale factor is 1/3)

$y = -4f(x)$: $y = -4x^2$ → vertically stretched (scale factor = 4) AND reflected in the x-axis.

$y = -\frac{1}{5}f(x)$ $y = -\frac{1}{5}x^2$ → vertically compressed (s.f. = 1/5) AND reflected in the x-axis

Exponential Function:

Parent Function: $f(x) = 2^x$ (we can use any base)

$y = 2f(x)$: $y = 2(2^x)$

$y = \frac{1}{3}f(x)$: $y = \frac{1}{3}(2^x)$

$y = -4f(x)$: $y = -4(2^x)$

$y = -\frac{1}{5}f(x)$ $y = -\frac{1}{5}(2^x)$

Summary:

$y = a \cdot f(x)$

a determines • $\overbrace{\text{stretches/compressions}}^{\text{sign doesn't matter.}}$ and • reflections in x-axis.

$|a| > 1$ → vertical stretch

$0 < |a| < 1$ → vertical compression

$a < 0$ (neg) → reflection in the x-axis.